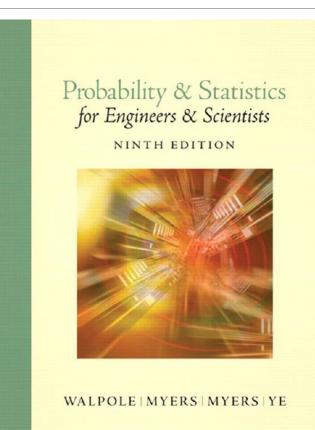
Statistical Analysis

Lecture 02

Books



PowerPoint

http://www.bu.edu.eg/staff/ahmedaboalatah14-courses/14767

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Agenda

>Review "The Central Limit Theorem"

Sampling Distribution of the Difference between Two Means

➤Sampling Distribution of S²

Sampling Distributions and Data Descriptions

CHAPTER 8

The Central Limit Theorem

Central Limit Theorem: If \overline{X} is the mean of a random sample of size n taken from a population with mean μ and finite variance σ^2 , then the limiting form of the distribution of

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}},$$

as $n \to \infty$, is the standard normal distribution n(z; 0, 1).

Ex 1:

1. Suppose the grades in a mathematics class are Normally distributed with a mean of 75 and a standard deviation of 6.

What is the probability that the average grade for 9 randomly selected students was at least 80?

Solution :

In this case, $\mu = 75$ and $\sigma = 6$.

We need to calculate the probability $P(\bar{X} > 80)$ with n = 9.

$$P(\overline{X} \ge 80) = P(\frac{X-\mu}{\frac{\sigma}{\sqrt{n}}} \ge \frac{80-75}{\frac{6}{\sqrt{9}}}) = P(Z \ge 2.5)$$

$$P(Z \ge 2.5) = 1 - P(Z < 2.5) = P(Z \le -2.5)$$

=0.0062

Sampling Distribution of the Difference between Two Means

Sampling Distribution of the Difference between Two Means

Suppose that we have two populations:

The first with mean μ_1 and variance σ_1^2 , and the second with mean μ_2 and variance σ_2^2 .

Let the statistic \bar{X}_1 represent the mean of a random sample of size n_1 selected from the first population, and

The statistic \bar{X}_2 represent the mean of a random sample of size n_2 selected from the second population, independent of the sample from the first population.

Sampling Distribution of the Difference between Two Means

According to Theorem 8.2, the variables X_1 and X_2 are both approximately normally distributed with means μ_1 and μ_2 and variances σ_1^2/n_1 and σ_2^2/n_2 , respectively. This approximation improves as n_1 and n_2 increase.

Sampling Distribution of the Difference between Two Means

By choosing independent samples from the two populations we ensure that the variables \bar{X}_1 and \bar{X}_2 will be independent, and then using Theorem 7.11, with $a_1 = 1$ and $a_2 = -1$, we can conclude that $\bar{X}_1 - \bar{X}_2$ is approximately normally distributed with mean

$$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_{\bar{X}_1} - \mu_{\bar{X}_2} = \mu_1 - \mu_2$$

and variance

$$\sigma_{\bar{X}_1-\bar{X}_2}^2 = \sigma_{\bar{X}_1}^2 + \sigma_{\bar{X}_2}^2 = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

Theorem 8.3:

If independent samples of size n_1 and n_2 are drawn at random from two populations, discrete or continuous, with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2 , respectively, then the sampling distribution of the differences of means, $\bar{X}_1 - \bar{X}_2$, is approximately normally distributed with mean and variance given by

$$\mu_{\bar{X}_1-\bar{X}_2} = \mu_1 - \mu_2 \text{ and } \sigma_{\bar{X}_1-\bar{X}_2}^2 = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

Hence,

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{(\sigma_1^2/n_1) + (\sigma_2^2/n_2)}}$$

is approximately a standard normal variable.

Case Study 8.2: Paint Drying Time:

Paint Drying Time: Two independent experiments are run in which two different types of paint are compared. Eighteen specimens are painted using type A, and the drying time, in hours, is recorded for each. The same is done with type B. The population standard deviations are both known to be 1.0.

Assuming that the mean drying time is equal for the two types of paint, find $P(\bar{X}_A - \bar{X}_B > 1.0)$, where \bar{X}_A and \bar{X}_B are average drying times for samples of size $n_A = n_B = 18$.

Solution: From the sampling distribution of $\bar{X}_A - \bar{X}_B$, we know that the distribution is approximately normal with mean

$$\mu_{\bar{X}_A - \bar{X}_B} = \mu_A - \mu_B = 0$$

and variance

$$\sigma_{\bar{X}_A-\bar{X}_B}^2 = \frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B} = \frac{1}{18} + \frac{1}{18} = \frac{1}{9}$$

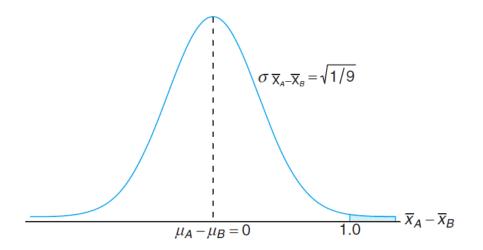


Figure 8.5: Area for Case Study 8.2.

The desired probability is given by the shaded region in Figure 8.5. Corresponding to the value $\bar{X}_A - \bar{X}_B = 1.0$, we have

$$z = \frac{1 - (\mu_A - \mu_B)}{\sqrt{1/9}} = \frac{1 - 0}{\sqrt{1/9}} = 3.0;$$

 \mathbf{SO}

P(Z > 3.0) = 1 - P(Z < 3.0) = 1 - 0.9987 = 0.0013.

15

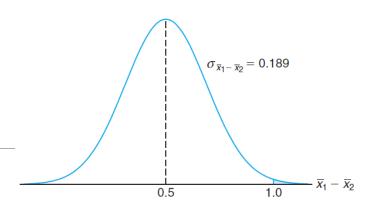
Example 8.6:

The television picture tubes of manufacturer A have a mean lifetime of 6.5 years and a standard deviation of 0.9 year, while those of manufacturer B have a mean lifetime of 6.0 years and a standard deviation of 0.8 year. What is the probability that a random sample of 36 tubes from manufacturer A will have a mean lifetime that is at least 1 year more than the mean lifetime of a sample of 49 tubes from manufacturer B?

Solution :

We are given the following information:

| Population 1 | Population 2 |
|------------------|------------------|
| $\mu_1 = 6.5$ | $\mu_2 = 6.0$ |
| $\sigma_1 = 0.9$ | $\sigma_2 = 0.8$ |
| $n_1 = 36$ | $n_2 = 49$ |



If we use Theorem 8.3, the sampling distribution of $\bar{X}_1 - \bar{X}_2$ will be approximately normal and will have a mean and standard deviation

$$\mu_{\bar{X}_1-\bar{X}_2} = 6.5 - 6.0 = 0.5$$
 and $\sigma_{\bar{X}_1-\bar{X}_2} = \sqrt{\frac{0.81}{36} + \frac{0.64}{49}} = 0.189.$

The probability that the mean lifetime for 36 tubes from manufacturer A will be at least 1 year longer than the mean lifetime for 49 tubes from manufacturer Bis given by the area of the shaded region in Figure 8.6. Corresponding to the value $\bar{x}_1 - \bar{x}_2 = 1.0$, we find that

$$z = \frac{1.0 - 0.5}{0.189} = 2.65,$$

and hence

$$P(\bar{X}_1 - \bar{X}_2 \ge 1.0) = P(Z > 2.65) = 1 - P(Z < 2.65)$$

= 1 - 0.9960 = 0.0040.

If a random sample of size n is drawn from a normal population with mean μ and variance σ^2 , and the sample variance is computed, we obtain a value of the statistic S^2 . We shall proceed to consider the distribution of the statistic $(n-1)S^2/\sigma^2$.

Start From :

$$\sum_{i=1}^n (X_i - \mu)^2 =$$

By the addition and subtraction of the sample mean \bar{X} , it is easy to see that

$$\sum_{i=1}^{n} (X_i - \mu)^2 = \sum_{i=1}^{n} [(X_i - \bar{X}) + (\bar{X} - \mu)]^2$$
$$= \sum_{i=1}^{n} (X_i - \bar{X})^2 + \sum_{i=1}^{n} (\bar{X} - \mu)^2 + 2(\bar{X} - \mu) \sum_{i=1}^{n} (X_i - \bar{X})^2$$
$$= \sum_{i=1}^{n} (X_i - \bar{X})^2 + n(\bar{X} - \mu)^2.$$

Dividing each term of the equality by σ^2 and substituting $(n-1)S^2$ for $\sum_{i=1}^n (X_i - \bar{X})^2$,

we obtain

$$\frac{1}{\sigma^2} \sum_{i=1}^n (X_i - \mu)^2 = \frac{(n-1)S^2}{\sigma^2} + \frac{(\bar{X} - \mu)^2}{\sigma^2/n}.$$

 $\sum_{i=1}^{n} \frac{(X_i - \mu)^2}{\sigma^2}$

The First term is a chi-squared random variable with n degrees of freedom. (X_1 to X_n and no relation in the equation)

$$\frac{(\bar{X}-\mu)^2}{\sigma^2/n}$$

The second term on the right-hand side is Z^2 , which is a chi-squared random variable with 1 degree of freedom. (only sample average)

$$\frac{(n-1)S^2}{\sigma^2}$$

The third term is a chi-squared random variable with n - 1 degree of freedom. (X_1 to X_n and relation in the equation between them (average))

Degrees of Freedom as a Measure of Sample Information

Example : suppose we have three numbers and we know their average is equal to zero then we have freedom to choose two of the three numbers

 $X_1 = 5$, $X_2 = 10$, and average = 0

But for the third one

There is no freedom to choose X_3

 $X_3 = 3^*$ average $-X_1 - X_2$

Degrees of Freedom as a Measure of Sample Information

as indicating that when μ is not known

considers the distribution of

$$\sum_{i=1}^n \frac{(X_i - \bar{X})^2}{\sigma^2},$$

there is 1 less degree of freedom, or a degree of freedom is lost in the estimation of μ (i.e., when μ is replaced by \bar{x}). In other words, there are *n* degrees of freedom, or independent *pieces of information*, in the random sample from the normal distribution. When the data (the values in the sample) are used to compute the mean, there is 1 less degree of freedom in the information used to estimate σ^2 .

Theorem 8.4:

If S^2 is the variance of a random sample of size *n* taken from a normal population having the variance σ^2 , then the statistic

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2} = \sum_{i=1}^n \frac{(X_i - \bar{X})^2}{\sigma^2}$$

has a chi-squared distribution with v = n - 1 degrees of freedom.

 χ^2_{α} represent the χ^2 value above which we find an area of α .

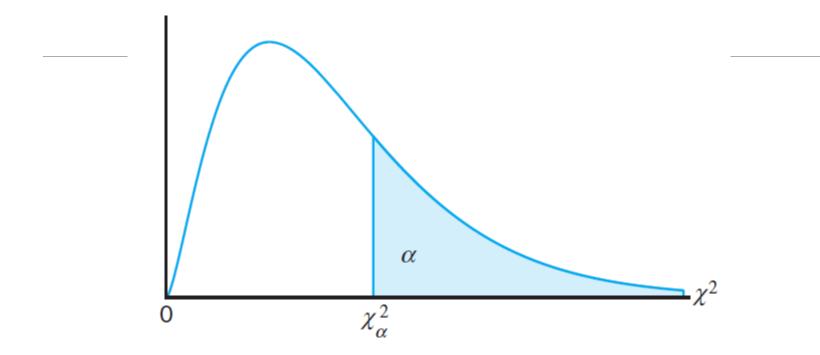
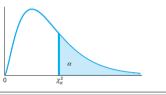


Table A.5 gives values of χ^2_{α} for various values of α and v. The areas, α , are the column headings; the degrees of freedom, v, are given in the left column; and the table entries are the χ^2 values. Hence, the χ^2 value with 7 degrees of freedom, leaving an area of 0.05 to the right, is $\chi^2_{0.05} = 14.067$. Owing to lack of symmetry, we must also use the tables to find $\chi^2_{0.95} = 2.167$ for v = 7.



 ${\bf Table \ A.5 \ Critical \ Values \ of \ the \ Chi-Squared \ Distribution}$

| = | | | | | | | | | | | |
|---|------------------|--------------|--------------|--------------|--------------|----------|--------|--------|--------|--------|--------|
| _ | | | | | | α | | | | | |
| | \boldsymbol{v} | 0.995 | 0.99 | 0.98 | 0.975 | 0.95 | 0.90 | 0.80 | 0.75 | 0.70 | 0.50 |
| - | 1 | $0.0^{4}393$ | $0.0^{3}157$ | $0.0^{3}628$ | $0.0^{3}982$ | 0.00393 | 0.0158 | 0.0642 | 0.102 | 0.148 | 0.455 |
| | 2 | 0.0100 | 0.0201 | 0.0404 | 0.0506 | 0.103 | 0.211 | 0.446 | 0.575 | 0.713 | 1.386 |
| | 3 | 0.0717 | 0.115 | 0.185 | 0.216 | 0.352 | 0.584 | 1.005 | 1.213 | 1.424 | 2.366 |
| | 4 | 0.207 | 0.297 | 0.429 | 0.484 | 0.711 | 1.064 | 1.649 | 1.923 | 2.195 | 3.357 |
| | 5 | 0.412 | 0.554 | 0.752 | 0.831 | 1.145 | 1.610 | 2.343 | 2.675 | 3.000 | 4.351 |
| | 6 | 0.676 | 0.872 | 1.134 | 1.237 | 1.635 | 2.204 | 3.070 | 3.455 | 3.828 | 5.348 |
| | 7 | 0.989 | 1.239 | 1.564 | 1.690 | 2.167 | 2.833 | 3.822 | 4.255 | 4.671 | 6.346 |
| | 8 | 1.344 | 1.647 | 2.032 | 2.180 | 2.733 | 3.490 | 4.594 | 5.071 | 5.527 | 7.344 |
| | 9 | 1.735 | 2.088 | 2.532 | 2.700 | 3.325 | 4.168 | 5.380 | 5.899 | 6.393 | 8.343 |
| 1 | 10 | 2.156 | 2.558 | 3.059 | 3.247 | 3.940 | 4.865 | 6.179 | 6.737 | 7.267 | 9.342 |
| 1 | 1 | 2.603 | 3.053 | 3.609 | 3.816 | 4.575 | 5.578 | 6.989 | 7.584 | 8.148 | 10.341 |
| 1 | 12 | 3.074 | 3.571 | 4.178 | 4.404 | 5.226 | 6.304 | 7.807 | 8.438 | 9.034 | 11.340 |
| 1 | 13 | 3.565 | 4.107 | 4.765 | 5.009 | 5.892 | 7.041 | 8.634 | 9.299 | 9.926 | 12.340 |
| 1 | L 4 | 4.075 | 4.660 | 5.368 | 5.629 | 6.571 | 7.790 | 9.467 | 10.165 | 10.821 | 13.339 |
| 1 | 15 | 4.601 | 5.229 | 5.985 | 6.262 | 7.261 | 8.547 | 10.307 | 11.037 | 11.721 | 14.339 |
| 1 | 16 | 5.142 | 5.812 | 6.614 | 6.908 | 7.962 | 9.312 | 11.152 | 11.912 | 12.624 | 15.338 |
| 1 | 17 | 5.697 | 6.408 | 7.255 | 7.564 | 8.672 | 10.085 | 12.002 | 12.792 | 13.531 | 16.338 |
| 1 | 18 | 6.265 | 7.015 | 7.906 | 8.231 | 9.390 | 10.865 | 12.857 | 13.675 | 14.440 | 17.338 |
| 1 | 19 | 6.844 | 7.633 | 8.567 | 8.907 | 10.117 | 11.651 | 13.716 | 14.562 | 15.352 | 18.338 |
| 2 | 20 | 7.434 | 8.260 | 9.237 | 9.591 | 10.851 | 12.443 | 14.578 | 15.452 | 16.266 | 19.337 |
| 2 | 21 | 8.034 | 8.897 | 9.915 | 10.283 | 11.591 | 13.240 | 15.445 | 16.344 | 17.182 | 20.337 |
| 2 | 22 | 8.643 | 9.542 | 10.600 | 10.982 | 12.338 | 14.041 | 16.314 | 17.240 | 18.101 | 21.337 |
| 2 | 23 | 9.260 | 10.196 | 11.293 | 11.689 | 13.091 | 14.848 | 17.187 | 18.137 | 19.021 | 22.337 |
| 2 | 24 | 9.886 | 10.856 | 11.992 | 12.401 | 13.848 | 15.659 | 18.062 | 19.037 | 19.943 | 23.337 |
| 2 | 25 | 10.520 | 11.524 | 12.697 | 13.120 | 14.611 | 16.473 | 18.940 | 19.939 | 20.867 | 24.337 |
| 2 | 26 | 11.160 | 12.198 | 13.409 | 13.844 | 15.379 | 17.292 | 19.820 | 20.843 | 21.792 | 25.336 |
| 2 | 27 | 11.808 | 12.878 | 14.125 | 14.573 | 16.151 | 18.114 | 20.703 | 21.749 | 22.719 | 26.336 |
| 2 | 28 | 12.461 | 13.565 | 14.847 | 15.308 | 16.928 | 18.939 | 21.588 | 22.657 | 23.647 | 27.336 |
| 2 | 29 | 13.121 | 14.256 | 15.574 | 16.047 | 17.708 | 19.768 | 22.475 | 23.567 | 24.577 | 28.336 |
| ê | 30 | 13.787 | 14.953 | 16.306 | 16.791 | 18.493 | 20.599 | 23.364 | 24.478 | 25.508 | 29.336 |
| 4 | 40 | 20.707 | 22.164 | 23.838 | 24.433 | 26.509 | 29.051 | 32.345 | 33.66 | 34.872 | 39.335 |
| 5 | 50 | 27.991 | 29.707 | 31.664 | 32.357 | 34.764 | 37.689 | 41.449 | 42.942 | 44.313 | 49.335 |
| 6 | 30 | 35.534 | 37.485 | 39.699 | 40.482 | 43.188 | 46.459 | 50.641 | 52.294 | 53.809 | 59.335 |
| | | | | | | | | | | | |

| 2 2.408 2.773 3.219 4.605 5.991 7.378 7.824 9.210 10.597 3 3 3.665 4.108 4.642 6.251 7.815 9.348 9.837 11.345 12.838 4 4.878 5.385 5.989 7.779 9.488 11.143 11.668 13.277 14.860 3 5 6.064 6.626 7.289 9.236 11.070 12.832 13.388 15.086 16.750 3 6 7.231 7.841 8.558 10.645 12.592 14.449 15.033 16.812 18.548 3 7 8.383 9.037 9.803 12.017 14.067 16.013 16.622 18.475 20.278 3 8 9.524 10.219 11.030 13.362 15.507 17.535 18.168 20.090 21.955 3 9 10.656 11.389 12.242 14.684 16.919 19.023 19.679 21.666 23.589 3 10 11.781 12.549 | | | | | | α | | | | | | |
|---|------------------|--------------|----------------|---------------------|-------------|----------|--------|--------|----------------|--------|--------|------------------|
| 2 2.408 2.773 3.219 4.605 5.991 7.378 7.824 9.210 10.597 3 3 3.665 4.108 4.642 6.251 7.815 9.348 9.837 11.345 12.838 4 4.878 5.385 5.989 7.779 9.488 11.143 11.668 13.277 14.860 3 5 6.064 6.626 7.289 9.236 11.070 12.832 13.388 15.086 16.750 3 6 7.231 7.841 8.558 10.645 12.592 14.449 15.033 16.812 18.548 3 7 8.383 9.037 9.803 12.017 14.067 16.013 16.622 18.475 20.278 3 8 9.524 10.219 11.030 13.362 15.507 17.535 18.168 20.090 21.955 3 9 10.656 11.389 12.242 14.684 16.919 19.023 19.679 21.666 23.589 3 10 11.781 12.549 | 0.001 | 0.005 0. | 0.01 0.00 | 0.02 0.01 | 0.025 0.0 | 0.02 | 0.05 | 0.10 | 0.20 | 0.25 | 0.30 | \boldsymbol{v} |
| 3 3.665 4.108 4.642 6.251 7.815 9.348 9.837 11.345 12.838 1 4 4.878 5.385 5.989 7.779 9.488 11.143 11.668 13.277 14.860 1 5 6.064 6.626 7.289 9.236 11.070 12.832 13.388 15.086 16.750 1 6 7.231 7.841 8.558 10.645 12.592 14.449 15.033 16.812 18.548 1 7 8.383 9.037 9.803 12.017 14.067 16.013 16.622 18.475 20.278 1 8 9.524 10.219 11.030 13.362 15.507 17.535 18.168 20.090 21.955 1 9 10.656 11.389 12.242 14.684 16.919 19.023 19.679 21.666 23.589 1 10 11.781 12.549 13.442 15.987 18.307 20.483 21.161 23.209 25.188 1 11 12.899 | 10.827 | 7.879 10.8 | 6.635 7.879 | 6.412 6 .635 | .024 5.412 | 5.024 | 3.841 | 2.706 | 1.642 | 1.323 | 1.074 | 1 |
| 4 4.878 5.385 5.989 7.779 9.488 11.143 11.668 13.277 14.860 14.850 14.8548 14.8548 14.8548 14.8548 14.8548 14.8548 14.8548 14.8548 14.8548 14.8548 14.864 16.919 19.023 19.679 21.666 23.589 14.966 14.830 14.830 14.830 14.830 14.830 14.631 17.275 19.675 | 13.815 | 10.597 13. | 9.210 10.597 | 7.824 9.210 | .378 7.824 | 7.378 | 5.991 | 4.605 | 3.219 | 2.773 | 2.408 | 2 |
| 5 6.064 6.626 7.289 9.236 11.070 12.832 13.388 15.086 16.750 2 6 7.231 7.841 8.558 10.645 12.592 14.449 15.033 16.812 18.548 2 7 8.383 9.037 9.803 12.017 14.067 16.013 16.622 18.475 20.278 2 8 9.524 10.219 11.030 13.362 15.507 17.535 18.168 20.090 21.955 2 9 10.656 11.389 12.242 14.684 16.919 19.023 19.679 21.666 23.589 2 10 11.781 12.549 13.442 15.987 18.307 20.483 21.161 23.209 25.188 2 11 12.899 13.701 14.631 17.275 19.675 21.920 22.618 24.725 26.757 2 12 14.011 14.845 15.812 18.549 21.026 23.337 24.054 26.217 28.300 2 13 < | 16.266 | 12.838 16.5 | 1.345 12.838 | 0.837 11.345 | .348 9.837 | 9.348 | 7.815 | 6.251 | 4.642 | 4.108 | 3.665 | 3 |
| 6 7.231 7.841 8.558 10.645 12.592 14.449 15.033 16.812 18.548 18.548 7 8.383 9.037 9.803 12.017 14.067 16.013 16.622 18.475 20.278 18.548 8 9.524 10.219 11.030 13.362 15.507 17.535 18.168 20.090 21.955 10.1781 9 10.656 11.389 12.242 14.684 16.919 19.023 19.679 21.666 23.589 10.1781 12.549 13.442 15.987 18.307 20.483 21.161 23.209 25.188 11.12.899 13.701 14.631 17.275 19.675 21.920 22.618 24.725 26.757 12.14.011 14.845 15.812 18.549 21.026 23.337 24.054 26.217 28.300 13.15.119 15.984 16.985 19.812 22.362 24.736 25.471 27.688 29.819 13.15.119 15.984 16.985 19.812 22.362 24.736 25.471 27.688 29.819 13.15.119 15.984 <td>18.466</td> <td>14.860 18.4</td> <td>3.277 14.860</td> <td>.668 13.277</td> <td>143 11.668</td> <td>11.143</td> <td>9.488</td> <td>7.779</td> <td>5.989</td> <td>5.385</td> <td>4.878</td> <td>4</td> | 18.466 | 14.860 18.4 | 3.277 14.860 | .668 13.277 | 143 11.668 | 11.143 | 9.488 | 7.779 | 5.989 | 5.385 | 4.878 | 4 |
| 7 8.383 9.037 9.803 12.017 14.067 16.013 16.622 18.475 20.278 2 8 9.524 10.219 11.030 13.362 15.507 17.535 18.168 20.090 21.955 2 9 10.656 11.389 12.242 14.684 16.919 19.023 19.679 21.666 23.589 2 10 11.781 12.549 13.442 15.987 18.307 20.483 21.161 23.209 25.188 2 11 12.899 13.701 14.631 17.275 19.675 21.920 22.618 24.725 26.757 2 12 14.011 14.845 15.812 18.549 21.026 23.337 24.054 26.217 28.300 2 13 15.119 15.984 16.985 19.812 22.362 24.736 25.471 27.688 29.819 3 | 20.515 | 16.750 20.5 | 5.086 16.750 | 15.086 | .832 13.388 | 12.832 | 11.070 | 9.236 | 7.289 | 6.626 | 6.064 | 5 |
| 8 9.524 10.219 11.030 13.362 15.507 17.535 18.168 20.090 21.955 2 9 10.656 11.389 12.242 14.684 16.919 19.023 19.679 21.666 23.589 2 10 11.781 12.549 13.442 15.987 18.307 20.483 21.161 23.209 25.188 2 11 12.899 13.701 14.631 17.275 19.675 21.920 22.618 24.725 26.757 2 12 14.011 14.845 15.812 18.549 21.026 23.337 24.054 26.217 28.300 2 13 15.119 15.984 16.985 19.812 22.362 24.736 25.471 27.688 29.819 2 | 22.457 | 18.548 22.4 | 5.812 18.548 | 6.033 16.812 | 449 15.033 | 14.449 | 12.592 | 10.645 | 8.558 | 7.841 | 7.231 | 6 |
| 9 10.656 11.389 12.242 14.684 16.919 19.023 19.679 21.666 23.589 2 10 11.781 12.549 13.442 15.987 18.307 20.483 21.161 23.209 25.188 2 11 12.899 13.701 14.631 17.275 19.675 21.920 22.618 24.725 26.757 2 12 14.011 14.845 15.812 18.549 21.026 23.337 24.054 26.217 28.300 2 13 15.119 15.984 16.985 19.812 22.362 24.736 25.471 27.688 29.819 2 | 24.321 | 20.278 24.3 | 8.475 20.278 | 6.622 	18.475 | 013 16.622 | 16.013 | 14.067 | 12.017 | 9.803 | 9.037 | 8.383 | 7 |
| 1011.78112.54913.44215.98718.30720.48321.16123.20925.18824.7251112.89913.70114.63117.27519.67521.92022.61824.72526.75726.7571214.01114.84515.81218.54921.02623.33724.05426.21728.30028.3001315.11915.98416.98519.81222.36224.73625.47127.68829.81929.819 | 26.124 | 21.955 26. | 0.090 - 21.955 | 3.168 20.090 | .535 18.168 | 17.535 | 15.507 | 13.362 | 11.030 | 10.219 | 9.524 | 8 |
| 1112.89913.70114.63117.27519.67521.92022.61824.72526.75726.7571214.01114.84515.81218.54921.02623.33724.05426.21728.30028.3001315.11915.98416.98519.81222.36224.73625.47127.68829.81929.819 | 27.877 | 23.589 27.5 | 1.666 - 23.589 | 0.679 21.666 | 023 19.679 | 19.023 | 16.919 | 14.684 | 12.242 | 11.389 | 10.656 | 9 |
| 12 14.011 14.845 15.812 18.549 21.026 23.337 24.054 26.217 28.300 3 13 15.119 15.984 16.985 19.812 22.362 24.736 25.471 27.688 29.819 3 | 29.588 | 25.188 29. | 3.209 25.188 | .161 23.209 | 483 21.161 | 20.483 | 18.307 | 15.987 | 13.442 | 12.549 | 11.781 | 10 |
| 13 15.119 15.984 16.985 19.812 22.362 24.736 25.471 27.688 29.819 32.471 27.688 29.819 27.471 27.688 29.819 27.681 29.819 27.681 27.68 | 31.264 | 26.757 31.5 | 4.725 26.757 | 2.618 24.725 | .920 22.618 | 21.920 | 19.675 | 17.275 | 14.631 | 13.701 | 12.899 | 11 |
| | 32.909 | 28.300 32.9 | 6.217 28.300 | .054 26.217 | .337 24.054 | 23.337 | 21.026 | 18.549 | 15.812 | 14.845 | 14.011 | 12 |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | 34.527 | 29.819 34. | 7.688 29.819 | 0.471 27.688 | 736 25.471 | 24.736 | 22.362 | 19.812 | 16.985 | 15.984 | 15.119 | 13 |
| | 36.124 | 31.319 36. | 9.141 31.319 | 6.873 29.141 | 119 26.873 | 26.119 | 23.685 | 21.064 | 18.151 | 17.117 | 16.222 | 14 |
| | 37.698 | 32.801 37. | 0.578 32.801 | 3.259 30.578 | 488 28.259 | 27.488 | 24.996 | 22.307 | 19.311 | 18.245 | 17.322 | 15 |
| 16 18.418 19.369 20.465 23.542 26.296 28.845 29.633 32.000 34.267 | 39.252 | 34.267 39.5 | 2.000 34.267 | 0.633 32.000 | .845 29.633 | 28.845 | 26.296 | 23.542 | 20.465 | 19.369 | 18.418 | 16 |
| 17 19.511 20.489 21.615 24.769 27.587 30.191 30.995 33.409 35.718 | 40.791 | 35.718 40.7 | 3.409 35.718 | .995 33.409 | 191 30.995 | 30.191 | 27.587 | 24.769 | 21.615 | 20.489 | 19.511 | 17 |
| 18 20.601 21.605 22.760 25.989 28.869 31.526 32.346 34.805 37.156 48.805 37.156 38.805 37.156 38.805 37.156 38.805 37.156 38.805 37.156 38.805 37.156 38.805 37.156 38.805 37.156 38.805 37.156 38.805 37.156 38.805 37.156 38.805 37.156 38.805 37.156 38.805 37.156 38.805 37.156 38.805 37.156 38.805 38.80 | 42.312 | 37.156 42.3 | 4.805 37.156 | 2.346 34.805 | 526 32.346 | 31.526 | 28.869 | 25.989 | 22.760 | 21.605 | 20.601 | 18 |
| 19 21.689 22.718 23.900 27.204 30.144 32.852 33.687 36.191 38.582 48.58 | 43.819 | 38.582 43. | 3.191 38.582 | 36.191 36.191 | .852 33.687 | 32.852 | 30.144 | 27.204 | 23.900 | 22.718 | 21.689 | 19 |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | 45.314 | 39.997 45.3 | 7.566 39.997 | 5.020 - 37.566 | 170 35.020 | 34.170 | 31.410 | 28.412 | 25.038 | 23.828 | 22.775 | 20 |
| 21 23.858 24.935 26.171 29.615 32.671 35.479 36.343 38.932 41.401 | 46.796 | 41.401 46.7 | 8.932 41.401 | 5.343 38.932 | 479 36.343 | 35.479 | 32.671 | 29.615 | 26.171 | 24.935 | 23.858 | 21 |
| | 48.268 | 42.796 48.5 | 0.289 42.796 | 7.659 40.289 | 781 37.659 | 36.781 | 33.924 | 30.813 | 27.301 | 26.039 | 24.939 | |
| 23 26.018 27.141 28.429 32.007 35.172 38.076 38.968 41.638 44.181 | 49.728 | 44.181 49. | 1.638 44.181 | .968 41.638 | .076 38.968 | 38.076 | 35.172 | 32.007 | 28.429 | 27.141 | 26.018 | 23 |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | 51.179 | 45.558 51. | 2.980 - 45.558 | 0.270 42.980 | .364 40.270 | 39.364 | 36.415 | 33.196 | 29.553 | 28.241 | 27.096 | 24 |
| 25 28.172 29.339 30.675 34.382 37.652 40.646 41.566 44.314 46.928 | 52.619 | 46.928 52. | 4.314 46.928 | .566 44.314 | 646 41.566 | 40.646 | 37.652 | 34.382 | 30.675 | 29.339 | 28.172 | 25 |
| 26 29.246 30.435 31.795 35.563 38.885 41.923 42.856 45.642 48.290 | 54.051 | 48.290 54.0 | 5.642 48.290 | 2.856 45.642 | .923 42.856 | 41.923 | 38.885 | 35.563 | 31.795 | 30.435 | 29.246 | 26 |
| 27 30.319 31.528 32.912 36.741 40.113 43.195 44.140 46.963 49.645 | 55.475 | 49.645 55.4 | 6.963 49.645 | .140 46.963 | 195 44.140 | 43.195 | 40.113 | 36.741 | 32.912 | 31.528 | 30.319 | 27 |
| | 56.892 | 50.994 56.8 | 8.278 50.994 | | | 44.461 | | 37.916 | | 32.620 | 31.391 | |
| | 58.301 | 52.335 58.3 | 9.588 - 52.335 | 6.693 49.588 | 722 46.693 | 45.722 | 42.557 | 39.087 | 35.139 | 33.711 | 32.461 | 29 |
| 30 33.530 34.800 36.250 40.256 43.773 46.979 47.962 50.892 53.672 40.256 43.773 46.979 47.962 50.892 53.672 40.256 40 | F0 700 | 52 672 50 | 0.892 53.672 | .962 50.892 | .979 47.962 | 46.979 | 43.773 | 40.256 | 36.250 | 34.800 | 33.530 | 30 |
| 40 44.165 45.616 47.269 51.805 55.758 59.342 60.436 63.691 66.766 | 59.702 | 55.072 59. | | 400 00 001 | a | 50.249 | 55 758 | 51 805 | 47 269 | 45 616 | 44 165 | 40 |
| | 59.702 73.403 | | 3.691 66.766 | 0.436 - 63.691 | .342 60.436 | -39.342 | 00.100 | 01.000 | H 1.203 | 10.010 | 44.100 | 40 |
| 60 65.226 66.981 68.972 74.397 79.082 83.298 84.58 88.379 91.952 9 | | 66.766 73.4 | | | | | | | | | | |

Table A.5 (continued) Critical Values of the Chi-Squared Distribution

Exactly 95% of a chi-squared distribution lies between $\chi^2_{0.975}$ and $\chi^2_{0.025}$. A χ^2 value falling to the right of $\chi^2_{0.025}$ is not likely to occur unless our assumed value of σ^2 is too small. Similarly, a χ^2 value falling to the left of $\chi^2_{0.975}$ is unlikely unless our assumed value of σ^2 is too large. In other words, it is possible to have a χ^2 value to the left of $\chi^2_{0.975}$ or to the right of $\chi^2_{0.025}$ when σ^2 is correct, but if this should occur, it is more probable that the assumed value of σ^2 is in error.

Example 8.7:

A manufacturer of car batteries guarantees that the batteries will last, on average, 3 years with a standard deviation of 1 year. If five of these batteries have lifetimes of 1.9, 2.4, 3.0, 3.5, and 4.2 years, should the manufacturer still be convinced that the batteries have a standard deviation of 1 year? Assume that the battery lifetime follows a normal distribution.

Solution :

We first find the sample variance using Theorem 8.1,

$$s^{2} = \frac{(5)(48.26) - (15)^{2}}{(5)(4)} = 0.815.$$

Then

$$\chi^2 = \frac{(4)(0.815)}{1} = 3.26$$

is a value from a chi-squared distribution with 4 degrees of freedom. Since 95% of the χ^2 values with 4 degrees of freedom fall between 0.484 and 11.143, the computed value with $\sigma^2 = 1$ is reasonable, and therefore the manufacturer has no reason to suspect that the standard deviation is other than 1 year.

